Radiative Transfer Theory to Estimate Medium Characteristics

Measure

scattering coefficient,
intrinsic attenuation

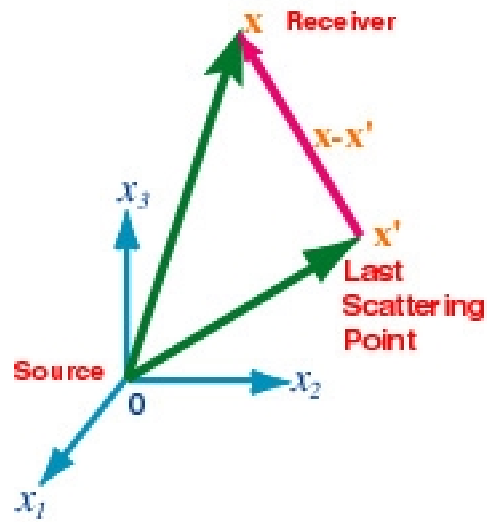
Terminology:

Q_i^{-1} Intrinsic Attenuation

Q_s^{-1} Scattering Attenuation

g Scattering Coefficient

$$g_0 \equiv \frac{1}{4\pi}\oint g d\Omega = \ell^{-1} = Q_s^{-1} k$$



Albedo: ratio of scattering to total attenuation

$$B_0 \equiv \frac{Q_S^{-1}}{Q_i^{-1} + Q_S^{-1}} = \frac{g_0 \beta_0}{g_0 \beta_0 + Q_i^{-1} \omega}$$

Terminology and Relationships:

Q_i^{-1} Intrinsic Attenuation

Q_s^{-1} Scattering Attenuation

Q_T^{-1} Total Attenuation

$$Q_T^{-1} = Q_S^{-1} + Q_i^{-1}$$

g Scattering Coefficient

$$g_0 \equiv \frac{1}{4\pi}\oint g\,d\Omega = \ell^{-1} = Q_s^{-1}k$$

B_o Albedo: ratio of scattering to total attenuation

$$B_0 \equiv \frac{Q_S^{-1}}{Q_i^{-1} + Q_S^{-1}} = \frac{g_0\beta_0}{g_0\beta_0 + Q_i^{-1}\omega}$$

First measurements done by Wu and Aki (1988)

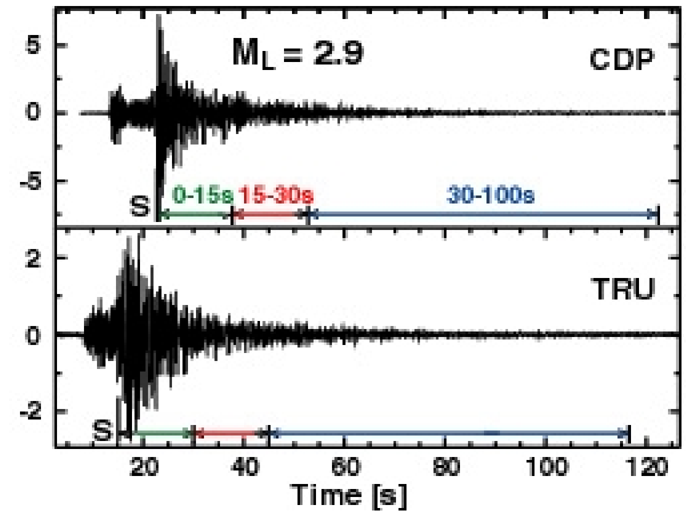
- Formulated radiative transfer theory for seismology
- Steady-state solution (Ishimaru, 1978)
- Measured wavefield energy *vs*. distance
  - compared observations with theory
  - Found scattering strength extremely strong

    $Q_s^{-1} >> Q_i^{-1}$

Multiple Lapse-Time Window Analysis Method

- Based on time-domain solution of radiative transfer theory equation
- Analyze Integrated Energy Density vs time and space
- Allows Separation of Scattering and Intrinsic Attenuation

Integrate Energy in Windows whose Times are Referenced to S-wave Arrival Time



Procedure

- Define 3 time windows and integrate energy in these windows

$$EI_1(f)_{kj} = \rho_0 \int_{0}^{15\text{s}} \left|\dot{u}_{kj}^{S}(t;f)\right|^2 dt,$$

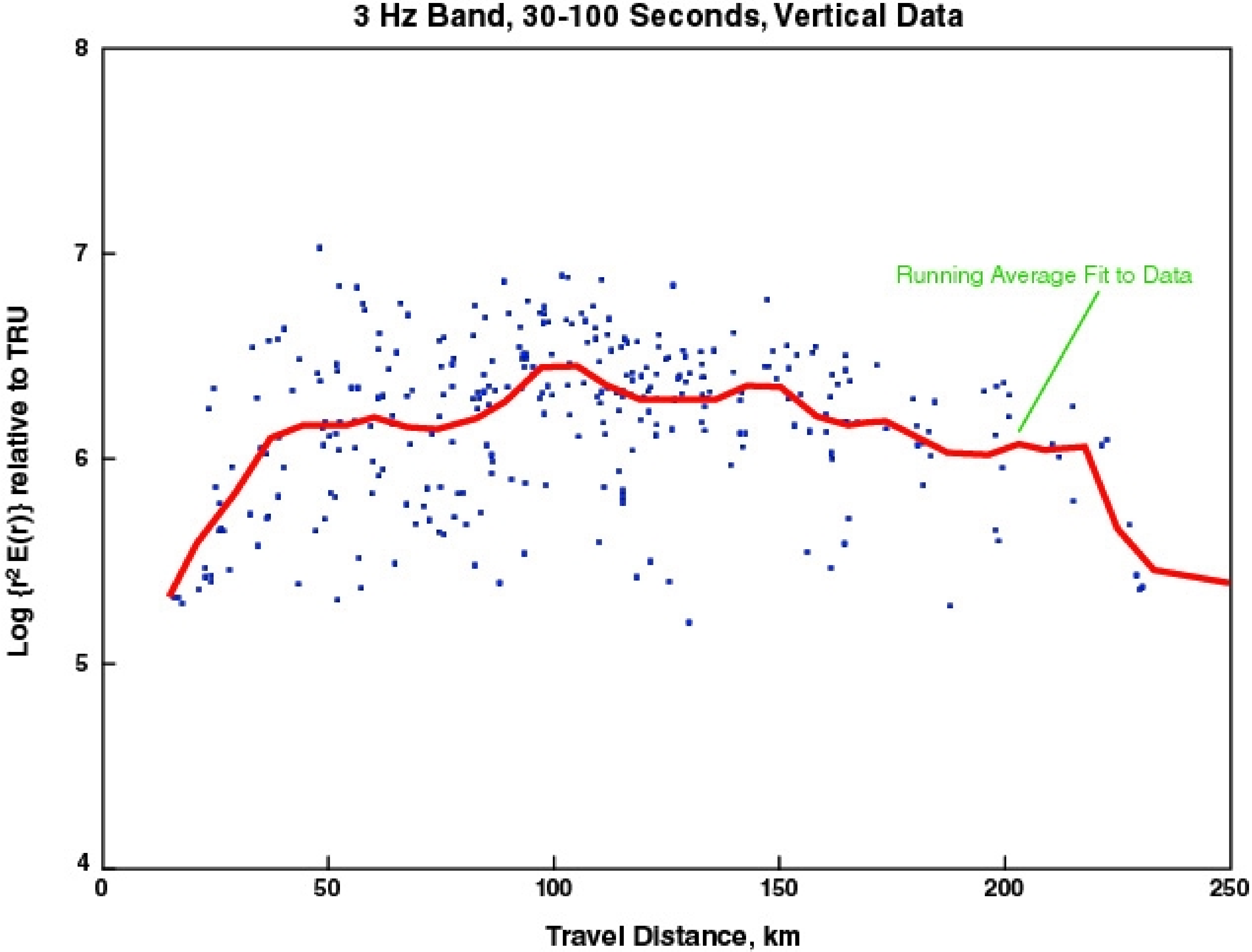
$$EI_2(f)_{kj} = \rho_0 \int_{15\text{s}}^{30\text{s}} \left|\dot{u}_{kj}^{S}(t;f)\right|^2 dt,$$

$$EI_3(f)_{kj} = \rho_0 \int_{30\text{s}}^{100\text{s}} \left|\dot{u}_{kj}^{S}(t;f)\right|^2 dt$$

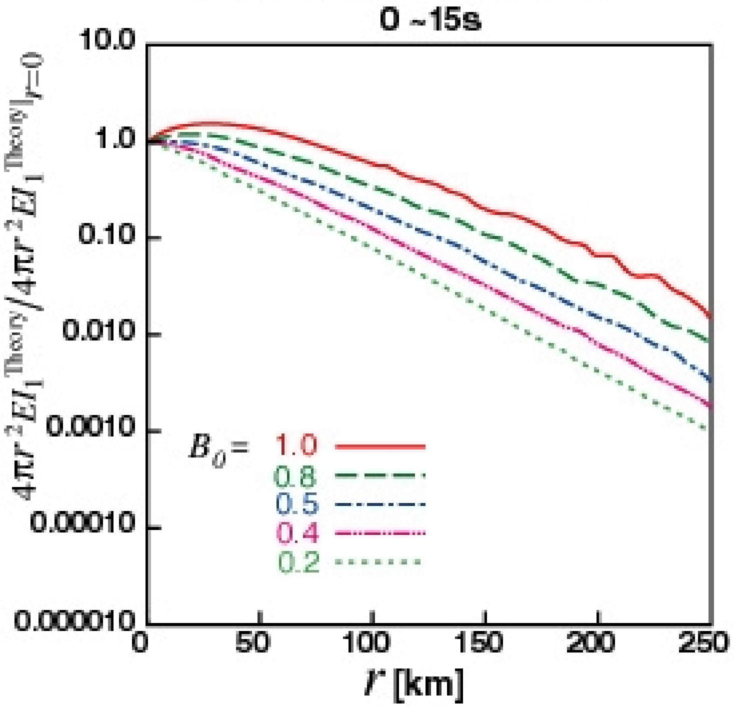
- Use Coda Normalization to correct data for station amplification and source size

Procedure #1

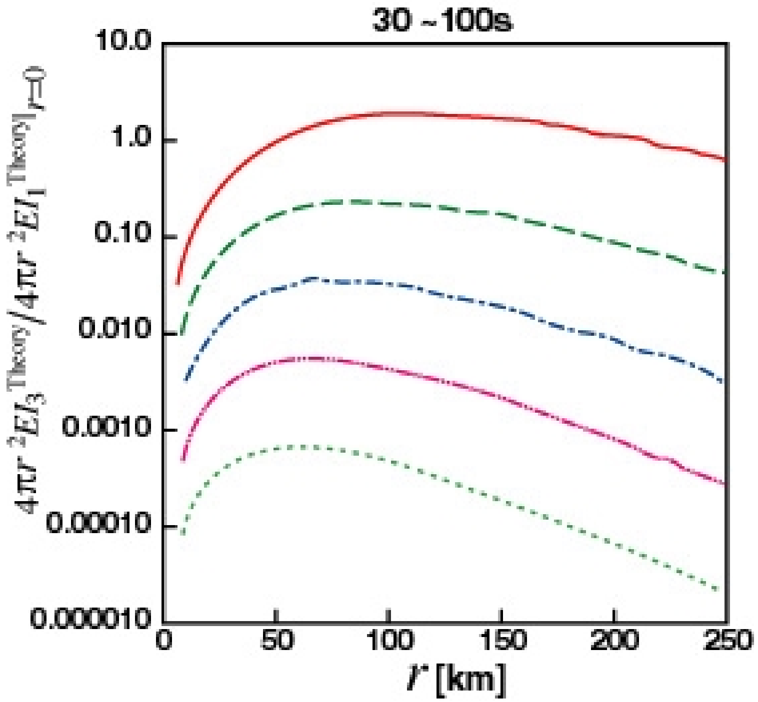
- Compare running average fits of data to model
- Comparing fits for three (or more?) time windows gives reasonably unique fit
- Could define "best" fit using some curve fitting procedure


3 Hz Band, 30-100 Seconds, Vertical Data
Running Average Fit to Data
Log {r² E(r)} relative to TRU
Travel Distance, km
8
7
6
5
4
0
50
100
150
200
250

Slope of Integrated Energy Vs Distance for First-Arrival Packet is Proportional to Total Attenuation



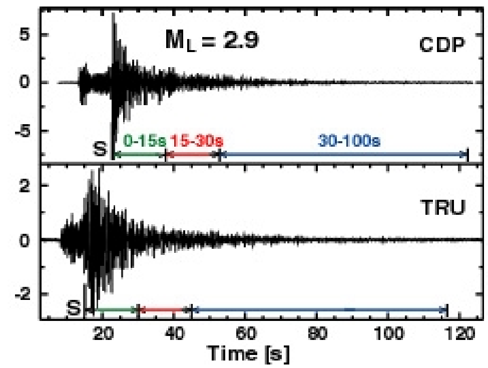
Amplitude of Integrated Energy Vs Distance for Late-Arriving Energy is Proportional to ScatteringAttenuation



Integrate Energy in Windows whose Times are Referenced to S-wave Arrival Time

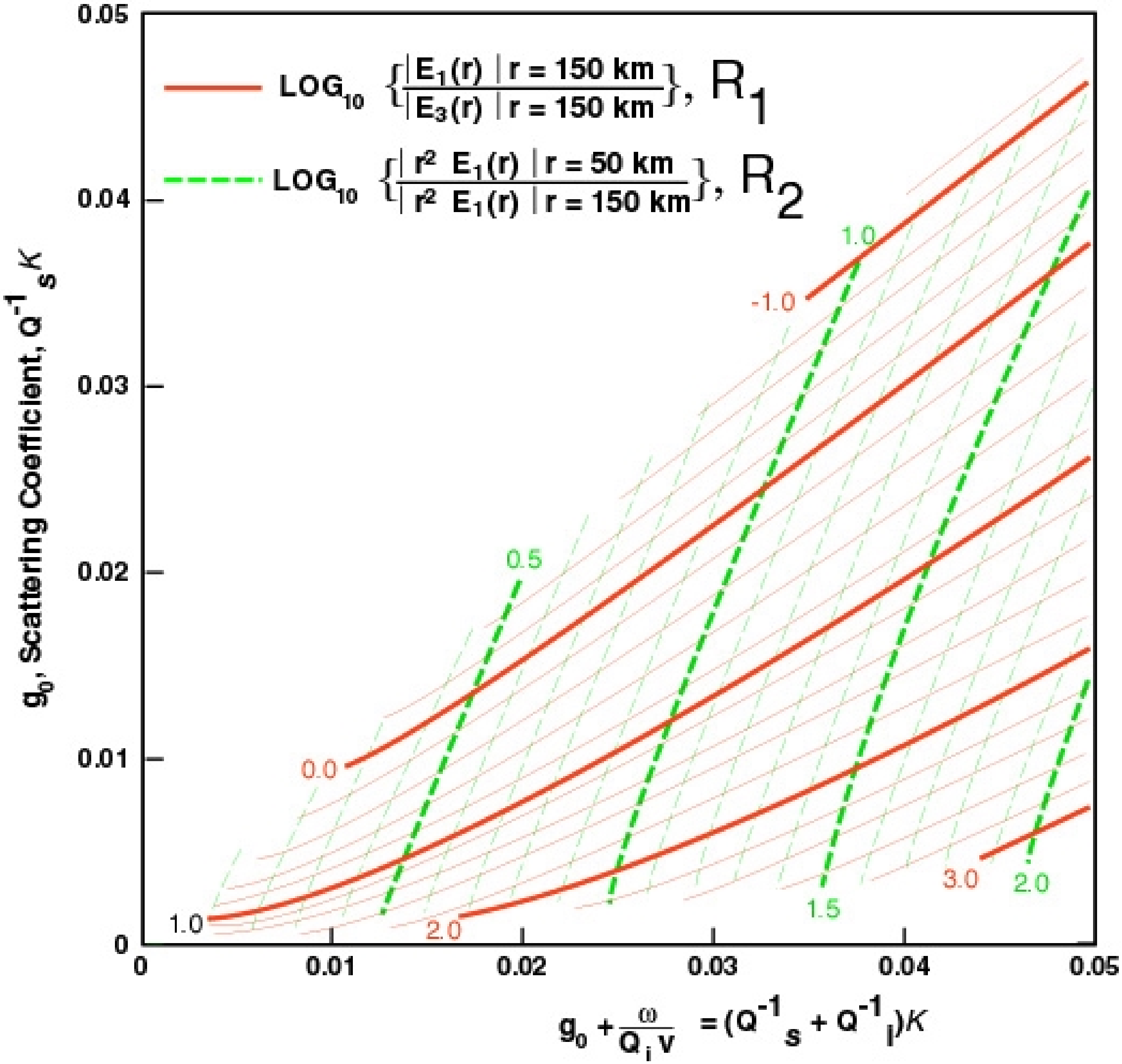
Procedure #2

- Measure following two



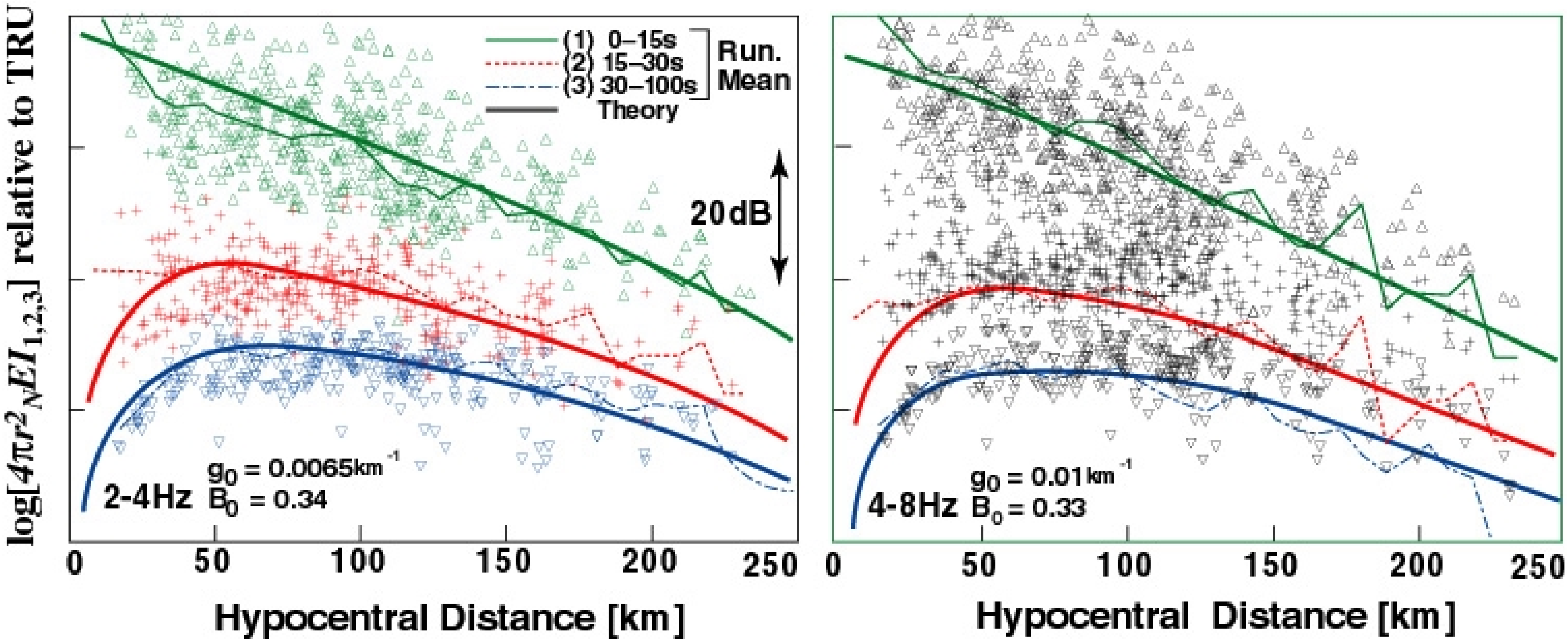
$$R_1 = \log\left[\frac{\left.\left\langle {}_N EI_1 \right\rangle_D\right|_{r=150\ \text{km}}}{\left.\left\langle {}_N EI_3 \right\rangle_D\right|_{r=150\ \text{km}}}\right] \qquad R_2 = \log\left[\frac{\left.4\pi r^2 \left\langle {}_N EI_1 \right\rangle_D\right|_{r=50\ \text{km}}}{\left.4\pi r^2 \left\langle {}_N EI_1 \right\rangle_D\right|_{r=150\ \text{km}}}\right]$$

- R_1 is proportional to amount of scattering
  - Large R_1 means no scattered waves
- R_2 is proportional to total attenuation
  - Slope of amplitude *vs* distance is classic way to determine total attenuation

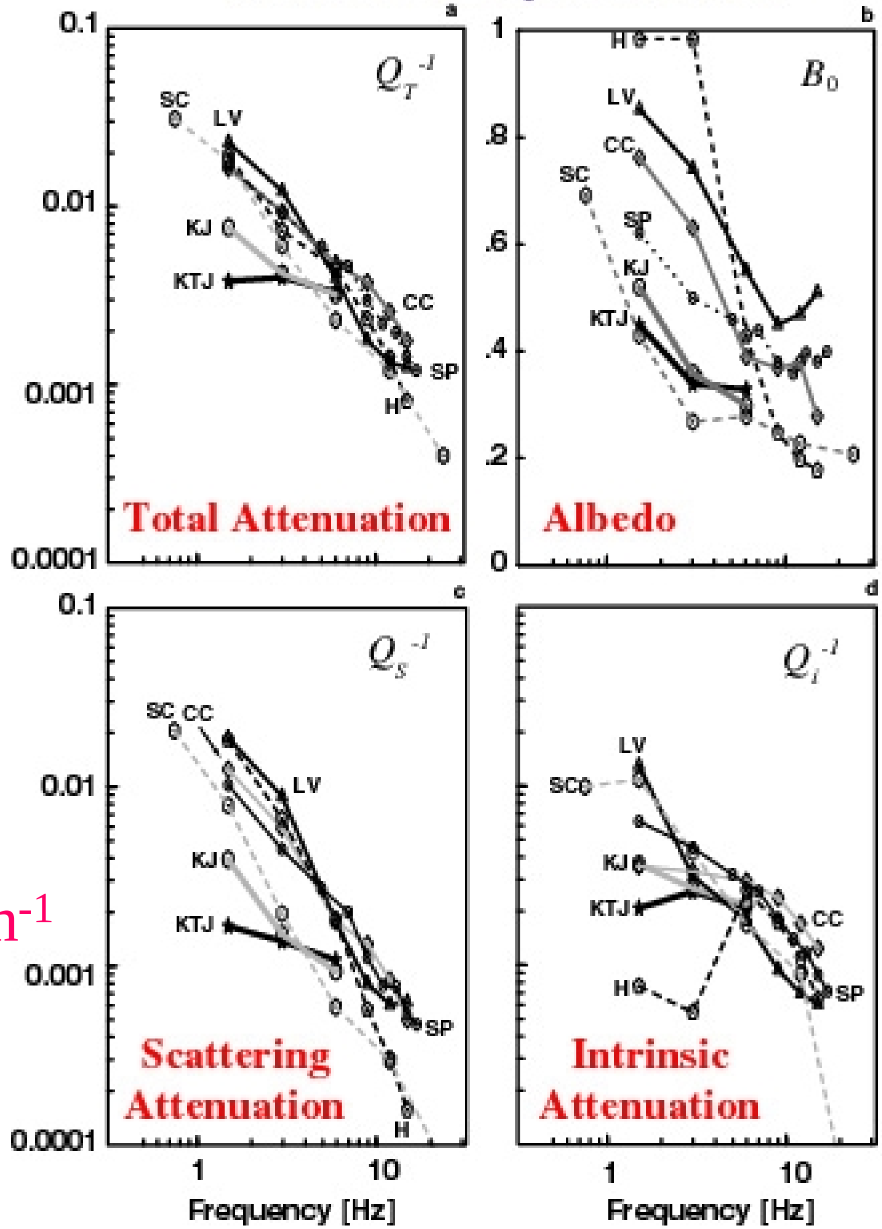


$\mathrm{LOG}_{10}\left\{\frac{|E_1(r)|\,r = 150\text{ km}}{|E_3(r)|\,r = 150\text{ km}}\right\}$, R_1
$\mathrm{LOG}_{10}\left\{\frac{|r^2\,E_1(r)|\,r = 50\text{ km}}{|r^2\,E_1(r)|\,r = 150\text{ km}}\right\}$, R_2
1.0
-1.0
0.5
0.0
1.0
2.0
1.5
3.0
2.0
g_0, Scattering Coefficient, $Q^{-1}{}_s K$
$g_0 + \frac{\omega}{Q_i v} = (Q^{-1}{}_s + Q^{-1}{}_I)K$
0
0.01
0.02
0.03
0.04
0.05

Fits to Data Collected in Japan



Results Obtained with Multiple Lapse-Time Window Analysis Method



Gives g_0 ~ .01 km^{-1}

- Feustel et al. (1996) studied mine seismicity (400-1600 Hz) and found $g_0 \sim .25\ m^{-1}$ which corresponds to characteristic length of mapped fractures
- Frequency dependence of g_0?

Other approaches for estimating medium characteristics:

- Fitting curves using optimization procedure
- Anisotropic scattering model

Conclusions

- Scattering models can be used to obtain useful information about the Earth
- Parameters vary with tectonic region and do provide information to characterize regions
- Coda normalization method works independently of any clear knowledge of its basis